# E–3824

## M. Sc./M. A. (Previous) EXAMINATION, 2021

MATHEMATICS

Paper Fourth

### (Complex Analysis)

*Time : Three Hours* ]                    [ *Maximum Marks : 100*

**Note :** All questions  are compulsory. Attempt any *two* parts from each Unit. All questions carry equal marks.

### Unit—I

1. (a)  State and prove Morera's theorem.

    (b)  State and prove Rouche's theorem.

    (c)  State and prove maximum modulus principle.

### Unit—II

2. (a)  State and prove Cauchy's residue theorem.

    (b)  Apply the calculus of Residue to prove that :

$$\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta}\,d\theta = \frac{\pi}{6}\,.$$

    (c)  State and prove Hurwitz's theorem.

### Unit—III

3. (a)  State and prove Mittag-Leffler's theorem.

    (b)  State and prove Schwarz's reflection principle.

(c) Show that there cannot be more than one continuation of an analytic function $f(z)$ into the same domain.

### Unit—IV

4. (a) State and prove Jensen's formula.

(b) State and prove Hadmard's three circles theorem.

(c) State and prove Borel's theorem.

### Unit—V

5. (a) State and prove Schottky's theorem.

(b) State and prove Montel Caratheodory theorem.

(c) Suppose $g$ is analytic on B (0 : R), $g$ (0) = 0 $|g'(0)| = u > 0$ and $|g(z)| \leq M$ for all $z$, then :

$$g(B(0; R)) \supset B\left(0; \frac{R^2 u^2}{6M}\right).$$